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Novel Hybrid Algorithm in Solving Unconstrained Optimizations Problems

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Abstract: **This paper introduces new hybrid algorithm between BFGS update of Hessian matrix in Quasi-Newton methods and coefficient conjugate of BZAU in CG methods. The convergence of hybrid algorithm has been proved by sufficient descent condition. In the end of this paper, numerical results demonstrate efficiency of new algorithm on standard test problems from previous studies.**

Keywords: **Unconstrained Optimization, Quasi-Newton algorithms, CG algorithms, BFGS update.**

I. INTRODUCTION

Unconstrained optimization problem [1] is given by

$$\min_{x \in R^n} f(x)$$

Where $f : R^n \to R$ is a twice-differential continuous function.

Iterative methods, used to solve unconstrained optimization problem presents approximate value to x in every iteration[1] according to the relation

$$x_{i+1} = x_i + \alpha_i d_i$$

d_i means direction search, and α_i means a step size. The direction search should achieve the relation of sufficient descent $g_i^T d_i \le 0$ in every iteration[1].

II. DIRECTION SEARCH

Different methods to solving the unconstrained optimization problems depends on calculating the direction search d_i. In this paper, The focus has been on two methods; CG methods and Quasi-Newton one.

CG method

The direction search in CG methods [1] is given by the following formula:

$$d_i = \begin{cases} -g_i & \text{if } i = 0; \\ -g_i + \beta_i d_{i-1} & \text{if } i \ge 1. \end{cases}$$

Where $g_i = \nabla f(x_i)$, and β_i defined as CG coefficient. There are many methods to calculate β_i such as [2-6].

Quasi-Newton methods

The direction search in Quasi-Newt methods [1] is given by solving the linear system

$$B_i d_i = -g_i$$

where $H_i = B_i^{-1}$ is Hessian matrix.

There is a few number of effective and wide used formula which are presented to converge Hessian matrix, such as SR1 [7], DFP [8], BFGS [9-12].

III. HYBRID ALGORITHM

A new hybrid direction search between BFGS update of Hessian matrix and CG conjugate coefficient β_i presented by Bakhtawar and others [13].

The direction search is given by the following relation:

$$d_i = \begin{cases} -H_i g_i & \text{if } i = 0; \\ -H_i g_i + \eta(-g_i + \beta_i^{BZAU} d_{i-1} - \theta_i^{BZAU} y_{i-1}) & \text{if } i \geq 1. \end{cases} \tag{1}$$

where $\eta > 0$ and β_i^{BZAU} is given by the formula:

$$\beta_i^{BZAU} = \frac{g_i^T(g_i - g_{i-1})}{-\zeta g_{i-1}^T d_{i-1} + \mu |g_i^T d_{i-1}|}$$

and θ_i^{BZAU} is given by formula:

$$\theta_i^{BZAU} = \frac{g_i^T d_{i-1}}{-\zeta g_{i-1}^T d_{i-1} + \mu |g_i^T d_{i-1}|}$$

Steps of algorithm

input: $x_0 \in R^n, H_0 = I, \epsilon$

step1: if $\nabla f(x_i) < \epsilon$ then stop

step2: compute d_i by (1)

step3: compute α_i by Strong Wolfe conditions [14].

step4: $x_{i+1} = x_i + \alpha_i d_i$

step5: update H_{i+1} with H_i by BFGS.

step6: i=i+1, go to step1

Theorem: The hybrid direction search implements the sufficient descent condition.

Prove: From relation, there is:

$$d_i = -H_i g_i + \eta(-g_i + \beta_i^{BZAU} d_{i-1} - \theta_i^{BZAU} y_{i-1})$$

Multiplying tow sides to g_i^T

$$g_i^T d_i = -g_i^T H_i g_i + \eta g_i^T(-g_i + \beta_i^{BZAU} - \theta_i^{BZAU} y_{i-1})$$

By substituting both of β_i^{BZAU} and θ_i^{BZAU}, the result will be as the following:

$$g_i^T d_i = -g_i^T H_i g_i + \eta(-g_i^T g_i + \frac{g_i^T(g_i - g_{i-1})(g_i^T d_{i-1})}{-\zeta g_{i-1}^T d_{i-1} + \mu|g_i^T d_{i-1}|} - \frac{(g_i^T d_{i-1}) g_i^T (g_i - g_{i-1})}{-\zeta g_{i-1}^T d_{i-1} + \mu|g_i^T d_{i-1}|}$$

Hence, the relation is obtained as the following

$$g_i^T d_i = -g_i^T H_i g_i + \eta(-g_i^T g_i)$$

$$g_i^T d_i \le -\lambda\|g_i\|^2 + \eta(-\|g_i\|^2)$$

$$g_i^T d_i \le \|g_i\|^2(-\lambda - \eta)$$

$$g_i^T d_i \le c\|g_i\|^2$$

where $c = (-\lambda - \eta)$

Sequentially, the sufficient descent condition will be as the following

$$g_i^T d_i \le 0$$

obtained.

IV. NUMERICAL RESULTS

This section presents applying the suggested hybrid method in solving the problem . Then, a comparison will by obtained between Quasi-Newt with (BFGS, DFP, SR1) updates and CG method with conjugate coefficient [15].

Applying this method will be based on standard function chosen from [15], adopting the following conditions and values:

Problems dimension $n = 2, \zeta = 1, \mu = 2, \eta = 10^{-5}$

Method	**Iterations**	$\|g_i\|$
Extended Freudenstein		
BFGS	22	0
DFP	151	0
SR1	f	f
CG	f	f
Hybrid method	14	0
Extended Trignometric		
BFGS	9	5.1884E-10
DFP	8	4.4567E-10
SR1	20	1.2457E-08
CG	6	2.6081E-08
Hybrid method	8	1.2280E-09
Extended Rosenbrock		
BFGS	42	2.1113E-10

Method	**Iterations**	$\|\|g_i\|\|$
DFP	46	1.2154E-10
SR1	62	8.9651E-09
CG	f	f
Hybrid method	33	3.4762E-10
Extended Beale		
BFGS	13	5.8664E-10
DFP	19	5.3557E-09
SR1	16	2.0388E-10
CG	21	9.7185E-07
Hybrid method	11	8.3111E-10
Perturbed Quadratic		
BFGS	7	1.4298E-10
DFP	7	8.6173E-12
SR1	4	1.3388E-14
CG	8	8.4177E-07
Hybrid method	6	1.1296E-12
Raydan 1		
BFGS	10	0
DFP	16	0
SR1	8	0
CG	6	0
Hybrid method	8	0
Raydan 2		
BFGS	6	0
DFP	6	0
SR1	f	f
CG	3	0
Hybrid method	4	0
Diagonal 2		
BFGS	9	0
DFP	9	0
SR1	16	0
CG	7	0
Hybrid method	6	0
Gen Tridiagonal 1		
BFGS	24	5.0943E-09
DFP	23	9.6471E-07
SR1	18	2.9933E-07
CG	10	5.8529E-07
Hybrid method	10	1.1984E-06

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Method	Iterations	$\|g_i\|$
Diagonal 5		
BFGS	4	0
DFP	4	0
SR1	5	0
CG	3	0
Hybrid method	3	0
Full Hessian FH1		
BFGS	27	4.8582E-07
DFP	33	2.1003E-07
SR1	26	5.8923E-07
CG	29	1.3198E-07
Hybrid method	18	1.8928E-06
Extended BD1		
BFGS	10	1.8279E-11
DFP	22	3.3602E-08
SR1	247	1.2188E-12
CG	20	5.2766E-07
Hybrid method	9	5.8952E-08
Extended Quadratic Penalty QP1		
BFGS	10	0
DFP	10	0
SR1	12	0
CG	22	0
Hybrid method	9	0
NONDIA		
BFGS	f	f
DFP	f	f
SR1	13	0
CG	f	f
Hybrid method	7	0
Broyden Tridiagonal		
BFGS	10	2.8878E-09
DFP	14	9.5626E-10
SR1	12	9.8992E-11
CG	26	4.2846E-07
Hybrid method	10	6.1991E-11
LIARWHD		
BFGS	24	2.9350E-11
DFP	58	4.3782E-11
SR1	28	1.5440E-11

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Method	**Iterations**	$\|g_i\|$
CG	f	f
Hybrid method	16	2.83572E-12
ENGVAL 1		
BFGS	10	0
DFP	10	0
SR1	f	f
CG	14	0
Hybrid method	8	0
SinQuad		
BFGS	20	1.6812E-07
DFP	21	2.8252E-07
SR1	230	1.9496E-07
CG	38	2.8333E-07
Hybrid method	12	4.9707E-05
Gen Quadratic		
BFGS	10	1.4735E-09
DFP	13	1.1908E-08
SR1	8	1.7593E-11
CG	7	3.1958E-10
Hybrid method	7	6.2678E-09
SinCos		
BFGS	14	0
DFP	14	0
SR1	12	0
CG	16	0
Hybrid method	9	0

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